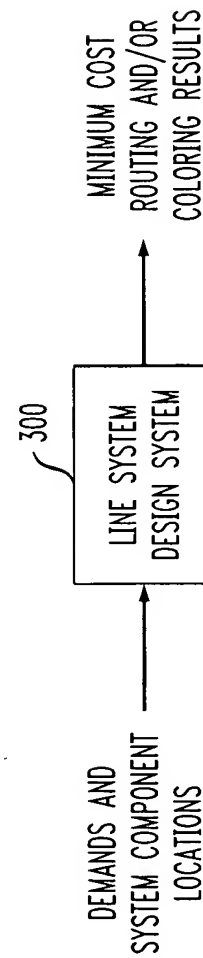


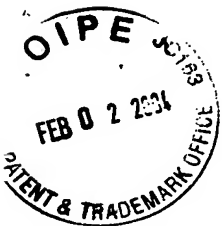


FIG. 2

| (A)          |                    |                    | (B)                 |            |
|--------------|--------------------|--------------------|---------------------|------------|
| GENERAL CASE |                    |                    | SPECIAL CASE        |            |
| PROBLEM      | APPROX LOWER BOUND | APPROX UPPER BOUND | PROBLEM             | COMPLEXITY |
| $(L, D, *)$  | $\Omega(\sqrt{s})$ | $O(\sqrt{s})$      | $(L, *, E), s = 2$  | POLYNOMIAL |
| $(L, U, NE)$ | $1 + 1/s^2$        | 2                  | $ C_2  = \infty$    |            |
| $(L, U, E)$  | NP-HARD            | 2                  | $(L, *, NE), s = 2$ | POLYNOMIAL |
| $(C, *, NE)$ | IN-APPROXIMABLE    |                    | $(L, U, E), s = 2$  | 4/3-APPROX |
| $(C, D, E)$  | IN-APPROXIMABLE    |                    | $(L, D, *), s = 3$  | NP-HARD    |
| $(C, U, E)$  | NP-HARD            | $2(1 + \epsilon)$  |                     |            |

FIG. 3





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FIG. 4A

Methodology A

```
 $m(0) = 0$ 
for  $p = 1$  to line system load/2
{
     $l(p) = 0$ ;  $m(p) = m(p-1) + 2$ ;
    for  $i = 1$  to  $n - 1$ 
    {
         $l_i(p) \leftarrow$  load on link  $e_i$ 
        if ( $l_i(p) = 0$ )
        {
            Divide the line system into two line systems;
            one from node 0 to node  $(i-1)$ ; the other from
            node  $i$  to node  $(n-1)$  and call methodology A
            on these line systems separately.
        }
        if ( $l_i(p) > l(p)$ )
        {
             $l(p) = l_i(p)$ 
        }
    }
    create a multigraph  $G = (V, E)$ , where  $V = \{0, \dots, n - 1\}$ 
    for all demand  $(i, j)$  in  $D$ 
    {
        create an edge  $(i - 1, j)$  in  $G$ 
    }
    for  $i = 1$  to  $n - 1$ 
    {
        if  $l_i(p) < l(p)$ 
            add an edge  $(i - 1, i)$  in  $G$ 
    }
    set the capacity of each edge in  $G$  to 1
    find a 2-unit flow from node 0 to node  $(n - 1)$  in  $G$ 
    Let  $p1$  and  $p2$  be the path for the flow
    For all the demands corresponding to links in  $p1$ .
    {
        Assign the color  $c_{m(p)}$  to demand
        remove the demand from  $D$ 
    }
    For all the demands corresponding to links in  $p2$ 
    {
        Assign the color  $c_{m(p)+1}$  to demand
        remove the demand from  $D$ 
    }
}
```



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FIG. 4B

Routing Phase:

if  $(L(R_s) \geq n(1 + \epsilon)/\epsilon)$

**Output**  $R_s$

else {

    Compute  $D_1 = \{d \in D \mid d \text{ in any routing goes through at least } n/3 \text{ links}\}$

    Compute  $D_2 = D - D_1$

    Compute  $R_1$  = the set of all possible routings for demands in  $D_1$

    Compute  $R_2$  = the set of all possible routings for demands in  $D_2$

    in which at most  $3S$  demands are not routed on shortest paths

    Compute  $R = R_1 \times R_2$

    Compute  $r \in R$  such that  $L(r) = \min_{r' \in R} L(r')$

**Output**  $r$

}

Coloring Phase:

$U = D$

$M$  = the set of available colors

$l = \min_{e_i \in L} l_i(U)$  (the min. load of demands in  $U$ )

while  $(l > 0)$  {

    Compute  $O = H(U)$  (see below)

    Compute  $m = \{i, j \mid i, j \text{ are the smallest two colors in } M\}$

    Color demands in  $O$  with colors in  $m$

$U = U - O$

$M = M - m$

$l = \min_{e_i \in L} l_i(U)$

}

if  $(U \neq \emptyset)$  {

    Color  $U$  using methodology A

“Compute  $O = H(U)$ ”:

    Compute  $d_0$  = a demand in  $U$  that goes through the largest number of links in  $L$

$O = \{d_0\}$

$L'$  = set of links covered by demands in  $O$

$i = 1$

    while  $(L' \neq L)$  {

        Compute  $D_i = \{d \mid d \in U - O \text{ \& } d \text{ overlaps with } d_{i-1}\}$

        Compute  $d_i = \{d \mid d \in D_i \text{ \& } d \text{ goes through the largest number of links in } L - L'\}$

$i = i+1$

**output**  $O$

}



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FIG. 4C

Methodology B:

$$e_0 = (-1, 0)$$

$$e_{n+1} = (n, n + 1)$$

$$L = L \cup \{e_0, e_{n+1}\}$$

$$D = D \cup \{(0, 0), (n + 1, n + 1)\}$$

for all  $(0 \leq i \leq j \leq n + 1)$  {

$$P(i, j) = \emptyset$$

$$R(i, j) = \emptyset$$

$$\text{best} = 0$$

for all  $(i \leq i' \leq j' \leq j)$  {

$$E_1 = \{e_i, e_{i+1}, \dots, e_{i'}\} \cup \{e_{j'+1}, e_{j'+2}, \dots, e_j\}$$

$$E_2 = e_{i'+1}, e_{i'+2}, \dots, e_{j'}$$

Compute coloring  $C$  using methodology b1 where  $E_1$  ( $E_2$ ) links are colored with 1 (2) steps

if  $(C \neq \emptyset)$  {

if  $(i' - i + j - j' + 1 \geq \text{best})$  {

$$R(i, j) = C$$

$$\text{best} = i' - i + j - j' + 1$$

}

}

}

}

Compute  $L_1 = \{e_i \mid e_i \in L, l_i \leq |C_1|\}$

for all  $(e_i, e_j \in L_1)$  {

Compute  $D_{i,j} = \{d \mid d \in D, d \text{ goes through either link } e_i, e_j\}$

Compute  $P_{i,j}$  = coloring obtained by coloring the interval graph  $D_{i,j}$  with colors in  $C_1$

}

for all  $(e_i, e_j \in L_1, i < j)$  {

$$\text{best} = 0$$

for all  $(m, i < m < j)$  {

Compute the coloring  $K = P(i, m) + P(m, j)$

If  $(K = \emptyset)$  continue

Compute  $n$  = number of links that are in one step in  $K$

if  $(\text{best} < n)$  {

$$\text{best} = n$$

$$C = K$$

}

}

Compute  $n$  = number of links that are in one step in  $R(i, j)$

if  $(\text{best} < n)$  {

$$\text{best} = n$$

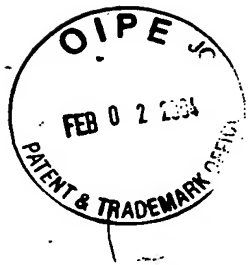
$$C = R(i, j)$$

}

$$P(i, j) = C$$

}

Output  $P(0, n + 1)$



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*FIG. 4D*

Methodology b1:

Compute  $C$  = interval graph coloring of demands  $D_1$  using colors in  $C_1$

if( $C == \emptyset$ ) **Output**  $C$ .

Compute  $C'$  = interval graph coloring of the demands in  $D - D_1$  using first available colors

**Output**  $C' \cup C$

*FIG. 4E*

Methodology c1:

$V = \{0, 1, \dots, n-1\}$

$E = \emptyset$

for all demands  $((i, j) \in D - D_1) \{$

$E = E \cup \{(i-1, j)\}$

Directed link  $(i-1, j)$  has unit capacity

$\}$

for all links  $(e_i \in L) \{$

$E = E \cup \{(i-1, i)\}$

Directed link  $(i-1, i)$  has capacity  $|C_1| + |C_2| - l_i$

$\}$

Graph  $G = (V, E)$

Compute maxFlow = Max. Flow  $f$  in  $G$  from node 0 to node  $n-1$

if(maxFlow <  $|C_2|$ ) **Output**  $\emptyset$

Compute  $F_1 = \{d \mid f \text{ puts zero flow on the edge } (i-1, j) \text{ where demand } d = (i, j)\}$

Compute  $F_1 = F_1 \cup D_1$

Compute  $K_1$  = coloring that colors demands in  $F_1$  with colors in  $C_1$  only using interval graph coloring

Compute  $K_2$  = coloring that colors demands in  $D - F_1$  with colors in  $C_2$  only using interval graph coloring

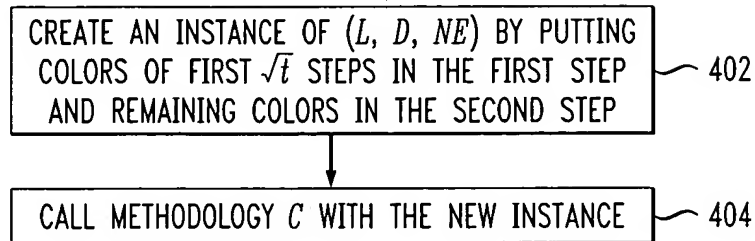
**Output**  $K = K_1 \cup K_2$



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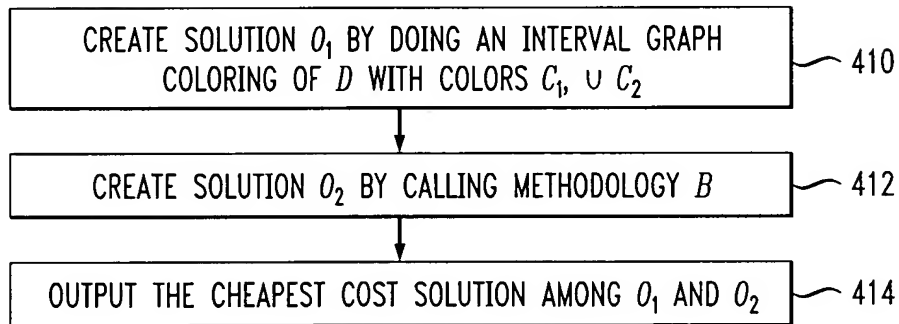
*FIG. 4F*

METHODOLOGY D:



*FIG. 4G*

METHODOLOGY E:



*FIG. 5*

